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A STATISTICAL ROSE PROGRAM

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ABSTRACT

This publication describes and evaluates a computer program useful for the statistical processing of bivariate data fields expressed in polar coordinates.

FOREWORD

An IBM 7070 computer program is described. This program is used by the U. S. Naval Oceanographic Office for the statistical processing of bivariate data fields expressed in rose form. The program logic is explained and the quality of the program is evaluated.

Scientific institutions desiring more specific information concerning this program are invited to write to the U. S. Naval Oceanographic Office.

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A STATISTICAL ROSE PROGRAM

I. INTRODUCTION

An IBM 7070 computer program is available which will calculate five parameters of frequency distribution over a two dimensional field (the components of the vector mean, the components of standard deviation along the principle axes of the distribution, and the inclination of the major axis), given either raw data, or vector sums and vector sums of squares of data, or processed roses. Having calculated the five parameters of distribution density, the program then integrates frequencies in the polar coordinates of the bivariate, tabulating frequencies for as many as six interval groups in eight cardinal directions, assuming normal distribution.

The program has several uses:

1. It may be used for the statistical processing of any kind of rose, including wind roses, sea and swell roses, current roses, cyclone roses, etc. Program application is not limited to velocity fields. The program may be applied to any bivariate field for which sampling distribution may be normal and elliptical.

2. The program may be used for the statistical processing of roses for which data are insufficient to conclude that a rose based on raw data is representative. The program calculates the parameters of distribution density associated with the data rose and produces a statistical rose with the same parameters. Assuming that the data sampling is random and sufficient for calculating representative parameters of distribution and that an infinite random data sampling would yield a normal elliptical frequency distribution field, the statistical rose may be regarded as representative for the rose station. In established study routines, statistical roses have been generated assuming that samples of twenty or more observations per station are adequate to yield representative parameter values but that samples of less than a hundred observations would not be adequate to yield a representative rose using standard methods.

3. Under certain circumstances the program may actually be used to calculate statistical roses for places where no data are available. Since the program is designed to calculate a statistical rose from five parameters of distribution density, these five parameters may be fed to the program as data to yield the corresponding statistical

rose. For an area with scattered stations one may interpolate by analysis procedures the five parameters to be used as data for those places with no data sampling.

4. The program may be adjusted to yield arbitrary speed-interval classes for the processed rose. The data rose may thus be processed for speed-interval classes which differ from those given with the rose. If, for example, the speed-interval breakdown of Weather Bureau data differs from a standard speed-interval format, such data may be processed to represent standard speed intervals by employing this program.

5. The program may be used to test the assumption graphically that large bivariate data samplings tend toward normalcy of distribution.

II. PROGRAM SPECIFICATIONS

The program was written in the IBM 7070 4-tape autocoder language so that maximum use could be made of efficient logic and fixed point arithmetic operations. The program occupies 3572 memory storage addresses at the time of this publication, including the IBM input-output subroutine package, and data fields.

The program generally requires twelve to fifteen seconds to process a rose. The frequency integration routine accounts for most of the time required for the production of a rose. Only a few milliseconds are required, in general, for calculating the distribution density parameters of a rose, assuming that the data rose input format is used. Since the machine rents for $2.50 per minute, machine cost for a single rose ranges between fifty and seventy cents.

Special routines written for the IBM 1401 are used for reading data cards and listing the completed rose.

III. PROGRAM LOGIC

A. Calculating the vector mean, vector variance and covariance:

1. "Raw" data: Given raw data consisting of two vector components per sample (cartesian or polar coordinates,) a subroutine may be employed which calculates the sum of components, the sum of the squares of components and the sum of component cross-products in the cartesian coordinate system.

Suppose that a raw data format is available which expresses wind data in degrees from north and knots. The subroutine sums and counts successive data samples and calculates the components of the vector mean, vector variance, and covariance according to the formulae:

$$v_{xj} = v_j \sin \theta_j \tag{1}$$

$$v_{yj} = v_j \cos \theta_j \tag{2}$$

$$\bar{v}_x = \frac{1}{N} \sum v_{xj} \tag{3}$$

$$\bar{v}_y = \frac{1}{N} \sum v_{yj} \tag{4}$$

$$S_x{}^2 = \frac{1}{N} \sum v_{xj}{}^2 - \bar{v}_x{}^2 \tag{5}$$

$$S_y{}^2 = \frac{1}{N} \sum v_{yj}{}^2 - \bar{v}_y{}^2 \tag{6}$$

$$\text{cov} = \frac{1}{N} \sum v_{xj} v_{yj} - \bar{v}_x \bar{v}_y \tag{7}$$

where
N: Total frequency of observations
j: The jth data sample
θ_j: Degrees from north of the jth data sample
v_j: speed (knots) of the jth data sample

x: The east-west coordinate axis
y: The north-south coordinate axis

$\bar{v}_x$: The X-component of the vector mean

$\bar{v}_y$: The Y-component of the vector mean

S_x^2: The X-component of the vector variance

S_y^2: The Y-component of the vector variance

cov: The covariance of the X-Y components

2. Data rose: Given a data rose, wherein raw data has previously been tabulated in specified speed-interval classes for the eight cardinal directions; the vector mean, vector variance and covariance may be calculated using the formulae:

$$\Sigma v_d = \Sigma a_i F_{id}$$

$$\Sigma v_d^2 = \Sigma a_i^2 F_{id} \qquad (d = N, NE, E, \ldots, NW)$$

$$\bar{v}_x = \frac{1}{N}\left\{(\Sigma v_E - \Sigma v_W) + 0.70711\left[(\Sigma v_{NE} - \Sigma v_{SW}) + (\Sigma v_{SE} - \Sigma v_{NW})\right]\right\}$$

$$\bar{v}_y = \frac{1}{N}\left\{(\Sigma v_N - \Sigma v_S) + 0.70711\left[(\Sigma v_{NE} - \Sigma v_{SW}) - (\Sigma v_{SE} - \Sigma v_{NW})\right]\right\}$$

$$\overline{v_x v_x} = \frac{1}{N}\left\{(\Sigma v_E^2 + \Sigma v_W^2) + 0.5\left[(\Sigma v^2_{NE} + \Sigma v^2_{SW}) + (\Sigma v^2_{SE} + \Sigma v^2_{NW})\right]\right\}$$

$$\overline{v_y v_y} = \frac{1}{N}\left\{(\Sigma v_N^2 + \Sigma v_S^2) + 0.5\left[(\Sigma v^2_{NE} + \Sigma v^2_{SW}) + (\Sigma v^2_{SE} + \Sigma v^2_{NW})\right]\right\}$$

$$\overline{v_x v_y} = \frac{1}{N}\left\{0.5\left[(\Sigma v^2_{NE} + \Sigma v^2_{SW}) - (\Sigma v^2_{SE} + \Sigma v^2_{NW})\right]\right\} \tag{9}$$

then

$$S_x^2 = \overline{v_x v_x} - \bar{v}_x^2$$

$$S_y^2 = \overline{v_y v_y} - \bar{v}_y^2$$

$$\text{cov} = \overline{v_x v_y} - \bar{v}_x \bar{v}_y \tag{10}$$

where i: the ith speed group in
d: the dth direction class

a_i: the midpoint speed in the ith speed group

F_{id}: the frequency of observations in the (i-d)th speed-direction group

Because the centroid of distribution in a speed-direction group is not generally coincident with the point of intersection of the group midpoint speed and the ray bisecting the octal direction class, this method of calculating distribution parameters may not yield wholly accurate results. This method characteristically exaggerates the distribution spread, for instance, producing an overestimate for the scalar mean velocity and the vector variance. Functions may be applied which will approximately correct for the interval errors intrinsic in this method; however, the program does not presently incorporate these correction functions.

B. Calculating the inclination of the principle axes of the distribution and the components of standard deviation along the principle axes of the distribution: The equations used for calculating the parameters of the distribution were derived from equations provided in "An Introduction to the Theory of Statistics."*

Let
$$K_1 = S_x^2 + S_y^2 \tag{11}$$

$$K_2 = S_x^2 - S_y^2 \tag{12}$$

$$L = (K_2^2 + 4\,\mathrm{cov}^2)^{\frac{1}{2}} \tag{13}$$

then
$$E_1 = \left[0.5(K_1 + L)\right]^{\frac{1}{2}} \tag{14}$$

$$E_2 = (K_1 - E_1^2)^{\frac{1}{2}} \tag{15}$$

* G. U. Yule and M. G. Kendall, "An Introduction to the Theory of Statistics," Edition 13; Charles Griffin & Company, Limited; London, 1949, chapter 12, pp. 227-232.

Where E_1 : The component of standard deviation along the major axis of the distribution

E_2 : The component of standard deviation along the minor axis of the distribution

Let θ' represent the angle included between the X-coordinate and one of the principal axes of the distribution. Then

$$\tan 2\theta' = 2\text{cov} / K_2$$

or

$$\theta' = 0.5 \arctan (2\text{cov} / K_2) \qquad (16)$$

If $K_2 > 0$, let $\theta = \theta'$; otherwise let $\theta = \theta' + 90°$. If $K_2 = 0$, then if $\text{cov} < 0$, $\theta = -45°$; otherwise $\theta = +45°$. Then θ identifies the major axis of the distribution.

C. Integrating frequencies for the normal surface within the limits of specified direction and speed classes

1. General description of the method of integration: The program regards the normal surface as a grid of 2,228 areas of known frequency. The grid is uniquely defined by the parameters of distribution density. The program locates the centroid of each such area in the polar coordinates of the bivariate and accordingly tabulates the known frequency in the appropriate speed-direction group.

2. Assumptions employed in deriving the grid of areas of known frequency: The normal surface may be visualized as a bell shaped mound with elliptical horizontal cross-sections, its axis of symmetry representing the ordinate which measures density of distribution frequency and perpendicular to the plane which measures vector variates. The equation for distribution density (the normal surface) is given* for a point (x,y) in cartesian coordinates coincident with the principal axes of the distribution:

$$\sigma_{x,y} = \frac{e^{-\frac{1}{2}\left[(x/E_1)^2 + (y/E_2)^2\right]}}{2\pi E_1 E_2} \qquad (17.a)$$

* Yule and Kendall, op. cit.

The exponent in equation (17.a) is elliptical in form. Let a cross-section of the distribution be identified with the parameter, χ, such that

$$\frac{x^2}{E_1^2} + \frac{y^2}{E_2^2} = \chi^2 \tag{18.a}$$

The parameter χ identifies an ellipse which intersects the major axis of the distribution at the point $E_1\chi$ and the minor axis at the point $E_2\chi$. χ, then, is a factor of proportionality relating elliptical cross-sections of the distribution, and is everywhere constant along the perimeter of an ellipse. It follows from equation (17.a) that the density of distribution is likewise constant at each point along the perimeter of an elliptical cross-section, and that it may be given:

$$\sigma\chi = \frac{e^{-\frac{1}{2}\chi^2}}{2\pi E_1 E_2} \tag{17.b}$$

Let the polar coordinates (R,ϕ) be defined such that $\frac{x}{R} = \cos\phi$; $\frac{y}{R} = \sin\phi$. Then $x = R\cos\phi$; $y = R\sin\phi$, and the ellipse, equation (18.a) may be expressed in the polar coordinates (R,ϕ):

$$R^2 = E_1^2 E_2^2 \chi^2 / (E_2^2 \cos^2\phi + E_1^2 \sin^2\phi) = \mathbb{E}^2 \chi^2 \tag{18.b}$$

and $\qquad x = \mathbb{E}\chi\cos\phi, \qquad y = \mathbb{E}\chi\sin\phi$

The frequency of distribution is given for a segment:

$$F_{x,y} = \int_y \int_x \sigma_{x,y}\, dx\, dy \tag{19.a}$$

It is convenient to integrate equation (19.a) between ellipses. Then the integral (19.a) may be transformed to the elliptical coordinates, (χ,ϕ):

$$F_{\chi,\phi} = \int_{\phi}\int_{\chi} \sigma_{\chi}\; J\begin{pmatrix} x & y \\ \chi & \phi \end{pmatrix} d\chi \; d\phi \tag{19.b}$$

where the Jacobian, $J\begin{pmatrix} x & y \\ \chi & \phi \end{pmatrix} = \frac{\partial x}{\partial \chi}\frac{\partial y}{\partial \phi} - \frac{\partial x}{\partial \phi}\frac{\partial y}{\partial \chi} = \mathbb{E}^2\chi$ (20)

so that $F_{\chi,\phi} = \int_{\phi}\int_{\chi} \sigma_{\chi}\; \mathbb{E}^2\,\chi\; d\chi\; d\phi$ (19.c)

Since χ and ϕ are mutually independent, functions involving χ only may be extracted from equation (19.c) as constants with respect to ϕ, and conversely. Then, integrating between the limits χ_1 and χ_2, ϕ_1 and ϕ_2:

$$F_{\chi,\phi} = \frac{1}{\pi E_1 E_2}\left[\int_{\chi_1}^{\chi_2} e^{-\frac{1}{2}\chi^2}\,\chi d\chi\right]\left[\int_{\phi_1}^{\phi_2} \frac{1}{2}\,\mathbb{E}^2\; d\phi\right] \tag{21}$$

$$= \frac{1}{2\pi}\left(e^{-\frac{1}{2}\chi_1^2} - e^{-\frac{1}{2}\chi_2^2}\right)\left[\arctan\left(\frac{E_1}{E_2}\tan\phi_2\right) - \arctan\left(\frac{E_1}{E_2}\tan\phi_1\right)\right] \tag{22}$$

Evaluating equation (22) between the limits ($\chi_1 = 0$, $\chi_2 = \chi$) and ($\phi_1 = 0$, $\phi_2 = 2\pi$) provides the frequency of distribution enclosed by the ellipse χ:

$$F(\chi) = 1 - e^{-\frac{1}{2}\chi^2} \tag{23}$$

Conversely, given the frequency of distribution, F, enclosed by an ellipse, to identify the χ parameter of the ellipse:

$$\chi = \left[-2\,\ln(1 - F)\right]^{\frac{1}{2}} \tag{24}$$

It may be noted that the ϕ integral in equation (21) provides the area of the sector of the unit ellipse which is subtended by the angle $(\phi_2 - \phi_1)$. The χ integral provides the frequency of distribution between the ellipses χ_1 and χ_2. The equation (21) therefore shows that sectors including equal areas of the unit ellipse likewise include equal frequencies of the distribution in the ring between adjacent ellipses. The normal elliptical distribution may therefore be subdivided into segments of known frequency, employing a grid of sectors subtending equal areas of the unit ellipse and ellipses enclosing known frequencies of the distribution.

The grid of sectors of equal area may be derived by employing an arbitrary parametric angle, ψ , such that points on the unit ellipse may be given in cartesian coordinates coincident with the principal axes of the distribution:

$$\bar{P} = (E_1 \cos\psi, E_2 \sin\psi) \tag{25}$$

accordingly, $$\tan\phi = \frac{E_2}{E_1} \tan\psi$$

so that the area of the sector defined by ψ is given:

$$A = \frac{E_1 E_2}{2} \arctan\left(\frac{E_1}{E_2} \tan\phi\right) = \frac{E_1 E_2}{2} \psi$$

and integer multiples of the parametric angle ψ shall define integer multiples of the unit sector area defined by ψ .

Given the grid of segments of known frequency, it is required to find a representative point in each segment such that if the segment overlaps adjacent speed-direction groups the point shall be tallied in the group containing the greatest portion of the segment frequency. The centroid of the segment may be regarded as the desired representative point. The coordinates of the centroid of a segment may be determined, in cartesian coordinates, (x,y), coincident with the principal axes of the distribution:

$$\bar{x} = \frac{\int_y \int_x x \, \sigma_{x,y} \, dx \, dy}{\int_y \int_x \sigma_{x,y} \, dx \, dy} \qquad \bar{y} = \frac{\int_y \int_x y \, \sigma_{x,y} \, dx \, dy}{\int_y \int_x \sigma_{x,y} \, dx \, dy} \tag{26}$$

Since $\quad x = E_1 \chi \cos\psi, \qquad y = E_2 \chi \sin\psi$
the equations (26) may be transformed to the elliptical coordinates, (χ, ψ):

$$\xi_x = \frac{\int_\psi \int_\chi E_1 \, \chi \cos\psi \; \sigma_\chi \, J\left(\frac{x}{\chi}\,\frac{y}{\psi}\right) d\chi \, d\psi}{\int_\psi \int_\chi \sigma_\chi \, J\left(\frac{x}{\chi}\,\frac{y}{\psi}\right) d\chi \, d\psi}$$

$$= \frac{E_1 \int_\psi \int_\chi \sigma_\chi \, \chi^2 \cos\psi \, d\chi \, d\psi}{\int_\psi \int_\chi \sigma_\chi \, \chi \, d\chi \, d\psi}$$

$$= E_1 \left.\frac{H(\chi)}{F(\chi)}\right]_\chi \frac{\int_\psi \cos\psi \, d\psi}{\int_\psi d\psi} \tag{27}$$

where $\left. H(\chi)\right]_\chi = \int_\chi e^{-\frac{1}{2}\chi^2} \chi^2 \, d\chi, \quad \left. F(\chi)\right]_\chi = \int_\chi e^{-\frac{1}{2}\chi^2} \chi \, d\chi$

Likewise,
$$\xi_y = E_2 \left.\frac{H(\chi)}{F(\chi)}\right]_\chi \frac{\int_\psi \sin\psi \, d\psi}{\int_\psi d\psi} \tag{28}$$

$F(\chi)$, evaluated between ellipses χ_{j-1} and χ_j provides the frequency of distribution between the two ellipses. The function, $H(\chi)$, may be evaluated between the same limits:

$$\left. H(\chi)\right]_{\chi_{j-1}}^{\chi_j} = \int_{\chi_{j-1}}^{\chi_j} e^{-\frac{1}{2}\chi^2} \chi^2 \, d\chi = \sum_{n=0}^{\infty} \frac{(-1)^n \left(\chi_j^{2n+3} - \chi_{j-1}^{2n+3}\right)}{2^n (2n+3)\, n!} \tag{29}$$

Let $(n - \frac{1}{2})\psi$, define successive rays including sectors of equal areas in the unit ellipse for n = 0, 1, 2, etc. Then integrating the equations (27) and (28) between the limits (χ_{j-1}), (χ_j) and $(n-\frac{1}{2})\psi$, $(n+\frac{1}{2})\psi$ provides the components, $\overline{P}_{j,n}$, of the centroid of the corresponding segment:

$$\overline{P}_{j,n} = \frac{1}{\psi}\frac{H(\chi)}{F(\chi)}\Bigg]_{\chi_{j-1}}^{\chi_j}\left\{E_1\left[\sin(n+\tfrac{1}{2})\psi - \sin(n-\tfrac{1}{2})\psi\right], E_2\left[\cos(n-\tfrac{1}{2})\psi - \cos(n+\tfrac{1}{2})\psi\right]\right\}$$

$$= (E_1\,\overline{\chi}_j \cos n\psi,\ \ E_2\,\overline{\chi}_j \sin n\psi) \qquad (30)$$

$$\text{where } \overline{\chi}_j = \frac{H(\chi)}{F(\chi)}\Bigg]_{\chi_{j-1}}^{\chi_j} G(\psi), \text{ and } G(\psi) = \frac{2\sin\frac{1}{2}\psi}{\psi}$$

That is, the ray defined by $n\psi$ bisects the area of the sector defined between the rays $(n-\frac{1}{2})\psi$ and $(n+\frac{1}{2})\psi$, and so bisects the frequency of distribution included in the sector. It may be noted that when n = 0, $\overline{P}_{j,o}$ is proportional to the module, E_1, of the major axis. When $n\psi$= 90°, $\overline{P}_{j,n}$ is proportional to the module, E_2 of the minor axis. The locus of $\overline{P}_{j,n}$, for j, ψ constants, is an ellipse.

The function, $G(\psi)$ is the factor providing correction for sector parallax. It may be noted that as the parametric angle, ψ, approaches the limit, 0, $G(\psi)$ approaches the limit, 1, so that the sector parallax error vanishes as the sector angle vanishes.

The parameters χ_j and $n\psi$ thus define the grid of segment centroids, and may be predetermined. The equation (30) relates the grid to the individual rose operated upon by the program. Grid choice is arbitrary. Grid segments were defined for this program so as to enclose discrete values of distribution frequency. The corresponding χ_j values were then calculated employing equations (25), for given n-intervals, and (24). The grid was chosen for segment definition such that segment areas would be of comparable magnitude and of similar shape, and such that maximum programming advantage could be taken of grid symmetry. The grid specifications are presented in table I.

3. Locating the segment of known frequency in the grid of direction-speed interval classes: The equation (30) expresses the point, $R_{j,n}$, in cartesian coordinates coincident with the principal axes of the distribution, and is transformed to components in the coordinate system of the bivariate with the equations:

$$X_{j,n} = (E_1 \cos\theta)(\bar{\lambda}_j \cos n\psi) - (E_2 \sin\theta)(\bar{\lambda}_j \sin n\psi) + \bar{v}_x$$

$$Y_{j,n} = (E_2 \cos\theta)(\bar{\lambda}_j \sin n\psi) + (E_1 \sin\theta)(\bar{\lambda}_j \cos n\psi) + \bar{v}_y \qquad (31)$$

The point is located within the limits, V_1 and V_2, of a designated speed interval class if

$$V_1^2 \leq (X^2_{j,n} + Y^2_{j,n}) < V_2^2$$

The point is located within the appropriate octal direction class according to the following logic:

		located octant
(1) If $\lvert X_{j,n} \rvert \tan 22\frac{1}{2}° > \lvert Y_{j,n} \rvert$, then:	(a) if $X_{j,n} > 0$,	EAST
	(b) otherwise,	WEST
(2) If $\lvert Y_{j,n} \rvert \tan 22\frac{1}{2}° > \lvert X_{j,n} \rvert$, then:	(a) if $Y_{j,n} > 0$,	NORTH
	(b) otherwise,	SOUTH
(3) Otherwise, if $X_{j,n} > 0$, then:	(a) if $Y_{j,n} > 0$,	NORTHEAST
	(b) otherwise,	SOUTHEAST
(4) Otherwise, $X_{j,n} < 0$, then:	(a) if $Y_{j,n} > 0$,	NORTHWEST
	(b) otherwise,	SOUTHWEST

IV. ROSE QUALITY EVALUATION

It was not practical to hand-calculate a rose for control results after the manner employed by the program. Hand-calculation of a single rose would have required several months to perform, and would at best have provided a check against the arithmetic of the program rather than a check against fundamental program logic. Rather, test data known or assumed to be normally distributed were processed by the program and the results were examined for correspondence with the test data.

A. Brooks' circular normal rose data

Brooks'* roses supplied from tables were used as one source of test data. These roses are theoretically characterized by normal-circular distribution and are useful for testing the program logic, equation (30), for the special case, $E_1 = E_2$. The table roses, however, are derived by graphical interpolation methods, and may not be exactly normal nor circular in fact. A representative test sample is provided in appendix I. The Brooks' data rose is listed along with the components of the vector and scalar means which theoretically identify it. The program image of the data rose is listed for easy comparison.

B. Program roses as test data

Theoretically, the program supplies roses which are elliptically and normally distributed. Consequently the program rose supplies data ideal for a test against itself. A representative sample test is provided in appendix II.

C. Analysis of test results: This test provides an index of rose degeneration due both to the method of rose frequency integration and to the method (Section III.A.2) for calculating the parameters of the data rose. The method of rose frequency integration may be expected to produce distortions in the image rose because of segment area overlap between adjacent speed-direction groups. However, the frequency tabulation error due to the overlapping of a segment between adjacent speed-direction groups may be either positive or negative for a group, and the collection of such errors tend in general to cancel

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* Brooks and Carruthers, "Handbook of Statistical Methods in Meteorology," M. O. 538. London, Her Majesty's Stationery Office, 1953; Chapter 11.

as the frequency population of the group is tallied. The method of frequency integration therefore probably does not account for consistent divergences between the data rose and its program image. Distortions due to this method may be likened to a "white noise" which is diminished in intensity as the frequency grid is divided into smaller and smaller segments. The method (Section III.A.2), on the other hand, may be expected to produce image rose values for the scalar mean and vector deviations, E_1 and E_2, which consistently exceed the corresponding values for the data rose.

In appendix I it may be noted that E_1 and E_2 are only approximately equivalent. This may be taken to indicate either that the object rose does not represent an exactly circular distribution or that the method for calculating the parameters of the data rose may thus distort the relationship between these parameters. The more probable alternative is that the object rose does not represent an exactly circular distribution; in appendix II, wherein a program rose is compared with its own image, it may be noted that the inclination of the major axis and the ratio of deviations between the principal axes closely coincide.

The distortion due to the method of calculating parameters of the data rose is, however, apparent in both the Brooks' and program rose samples; that is, the program image values for the scalar mean velocity and the vector deviations exceed the corresponding parameters for the test rose. Nevertheless, corresponding values between the test roses and their program images are clearly similar, particularly for the program rose. Such similarity suggests that the program logic is successful, and that rose distortion due to segment overlap error is small. Unfortunately, the precise extent of segment overlap error is masked by the distortion which may be attributed to the method for calculating data rose parameters. The error introduced by the program method for frequency integration cannot be estimated with any precision from these test results.

D. Conclusion: The program can probably be improved by applying an interval correction function to the method (Section III.A.2) for calculating the parameters of a data rose. Doubtless, too, there is a more efficient method for integrating frequencies of the distribution in the speed-direction interval classes. However, test samples suggest that the program rose is of substantial quality, and that the program may be employed confidently whenever program application is warranted.

Appendix I: BROOKS' TEST DATA ROSE

Speed groups: I, 00 -10; II, 10 - 20; III, 20 - 40;
(knots) IV, 40 -60; V, 60 - 100; VI, 100+

	Test rose	program image	diff
mean	26.95	29.00	2.05
Vx	-19.10	-19.54	.44
Vy	- 6.60	- 6.50	.10
sinΘ	-	-.99625 *	-
E1	-	18.04	-
E2	-	16.85	-
N I	.80	.84	.04
II	1.40	1.52	.12
III	1.50	1.77	.28
IV	.30	.29	.01
V	0	0	0
VI	0	0	0
NE I	.50	.61	.11
II	.80	.76	.04
III	.50	.53	.03
IV	0	0	0
V	0	0	0
VI	0	0	0
E I	.50	.56	.06
II	.70	.78	.08
III	.40	.45	.05
IV	0	0	0
V	0	0	0
VI	0	0	0
SE I	.70	.65	.05
II	1.20	1.22	.02
III	1.00	1.18	.18
IV	.10	.14	.04
V	0	0	0
VI	0	0	0

	Test rose	program image	diff
S I	1.10	1.08	.02
II	2.80	2.92	.12
III	5.20	5.56	.36
IV	1.20	1.59	.39
V	0	.14	.14
VI	0	0	0
SW I	1.40	1.38	.02
II	5.50	5.07	.43
III	17.90	15.67	2.23
IV	6.60	7.35	.75
V	.60	.95	.35
VI	0	0	0
W I	1.40	1.38	.02
II	5.90	5.35	.55
III	19.30	17.42	1.88
IV	7.00	7.58	.58
V	.60	.93	.33
VI	0	0	0
NW I	1.20	1.14	.06
II	3.30	3.24	.06
III	7.00	7.38	.38
IV	1.80	2.35	.55
V	.10	.21	.11
VI	0	0	0
TOTAL I	7.60	7.64	.04
II	21.60	20.86	.74
III	52.80	49.96	2.84
IV	17.00	19.31	2.31
V	1.30	2.29	.99
VI	0	0	0

* The angle Θ is in the fourth quadrant

Appendix II: PROGRAM TEST DATA ROSE

Speed groups: I, 00 - 10; II, 10 - 20; III, 20 - 40;
(knots) IV, 40 - 60; V, 60 - 100; VI, 100+

	Test rose	program image	diff
mean	29.07	30.18	1.11
Vx	-14.71	-14.77	.06
Vy	10.49	10.52	.03
sinθ	.73534*	.73973*	.00439
E1	20.16	21.30	1.14
E2	18.29	19.38	1.09
E1/E2	1.102	1.099	.003
	******************		
N I	1.20	1.08	.12
II	3.60	3.24	.36
III	9.15	8.93	.22
IV	3.80	4.21	.41
V	.52	.74	.22
VI	0	0	0
		* * *	
NE I	.96	.90	.06
II	2.07	2.04	.03
III	3.42	3.54	.12
IV	.86	1.16	.30
V	.07	.14	.07
VI	0	0	0
		* * *	
E I	.72	.72	0
II	1.18	1.26	.08
III	1.08	1.34	.26
IV	.15	.18	.03
V	0	0	0
VI	0	0	0
		* * *	
SE I	.78	.72	.06
II	.97	1.05	.08
III	.71	.94	.23
IV	.10	.11	.01
V	0	0	0
VI	0	0	0
		* * *	

	Test rose	program image	diff
S I	.78	.66	.12
II	1.39	1.49	.10
III	1.61	1.80	.19
IV	.24	.33	.09
V	0	0	0
VI	0	0	0
		* * *	
SW I	1.08	1.08	0
II	2.58	2.40	.18
III	5.21	5.38	.17
IV	1.70	2.09	.39
V	.20	.34	.14
VI	0	0	0
		* * *	
W I	1.26	1.08	.18
II	4.32	3.96	.36
III	12.12	11.46	.66
IV	5.95	6.18	.23
V	.92	1.34	.42
VI	0	0	0
		* * *	
NW I	1.38	1.32	.06
II	4.88	4.47	.41
III	14.88	13.55	1.33
IV	7.07	7.21	.14
V	1.09	1.57	.48
VI	0	0	0
	******************		
TOTAL I	8.16	7.56	.60
II	20.99	19.91	1.08
III	48.18	46.94	1.24
IV	19.87	21.41	1.54
V	2.80	4.18	1.38
VI	0	0	0
		* * *	

* Both angles are in the first quadrant

TABLE 1: FREQUENCY GRID SPECIFICATIONS

Ellipse number, j	sector angle, $\psi°$	num of segments ring $j-1,j$	FREQ (%) per segment	FREQ (%) ring $j-1,j$	cum num of segments	CUM FREQ in ellipse χ_j	χ_j	$H(\chi_j)$	$\bar{\chi}_j$
1	90	4	.04	.16	4	.16	0.056595	0.000060	0.036767
2	45	8	.05	.40	12	.56	0.105980	0.000395	0.081634
3	45	8	.05	.40	20	.96	0.138899	0.000888	0.120020
4	22.5	16	.05	.80	36	1.76	0.188450	0.002207	0.163830
5	22.5	16	.06	.96	52	2.72	0.234847	0.004247	0.211092
6	11.25	32	.06	1.92	84	4.64	0.308255	0.009490	0.272643
7	11.25	32	.06	1.92	116	6.56	0.368976	0.016001	0.338560
8	11.25	32	.06	1.92	148	8.48	0.420981	0.023588	0.394539
9	11.25	32	.06	1.92	180	10.40	0.468646	0.032135	0.444442
10	11.25	32	.06	1.92	212	12.32	0.512789	0.041562	0.490185
11	11.25	32	.06	1.92	244	14.24	0.554288	0.051809	0.532876
12	11.25	32	.06	1.92	276	16.16	0.593794	0.062833	0.573244
13	11.25	32	.06	1.92	308	18.08	0.631549	0.074599	0.611781
14	5.625	64	.06	3.84	372	21.92	0.703472	0.100245	0.667867
15	5.625	64	.06	3.84	436	25.76	0.771838	0.128580	0.737597
16	5.625	64	.06	3.84	500	29.60	0.837827	0.159491	0.804665
17	5.625	64	.06	3.84	564	33.44	0.902299	0.192905	0.869805
18	5.625	64	.06	3.84	628	37.28	0.965908	0.228776	0.933773
19	5.625	64	.06	3.84	692	41.12	1.029241	0.267089	0.997176
20	5.625	64	.06	3.84	756	44.96	1.092804	0.307825	1.060561
21	5.625	64	.06	3.84	820	48.80	1.157092	0.351020	1.124518
22	5.625	64	.06	3.84	884	52.64	1.222614	0.396706	1.189250
23	5.625	64	.06	3.84	948	56.48	1.289922	0.444940	1.255586
24	2.8125	128	.06	7.68	1076	64.16	1.432554	0.549401	1.360049
25	2.8125	128	.06	7.68	1204	71.84	1.592023	0.665407	1.510344
26	2.8125	128	.05	6.40	1332	78.24	1.746480	0.772109	1.666961
27	2.8125	128	.05	6.40	1460	84.64	1.935667	0.889685	1.837097
28	2.8125	128	.03	3.84	1588	88.48	2.078983	0.966667	2.004547
29	2.8125	128	.03	3.84	1716	92.32	2.326393	1.072847	2.193594
30	2.8125	128	.02	2.56	1844	94.88	2.438039	1.109978	2.379966
31	2.8125	128	.02	2.56	1972	97.44	2.707458	1.175505	2.559398
32	2.8125	128	.01	1.28	2100	98.72	2.952392	1.211570	2.817271
33	2.8125	128	.01	1.28	2228	100.00	∞	1.259314	3.260966

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